TRAJECTORY CONSIDERATIONS FOR CIRCUMLUNAR MISSIONS

William H. Michael, Jr. and Jack W. Crenshaw
NASA Langley Research Center
IAS Paper #61-35
Presented at the Inst. Aerospace Sci. 29th Annual Meeting,
New York, January 1961

SUMMARY

Current interest in circumlunar trajectories is stimulated by the fact that the circumlunar mission will probably be the first type of manned lunar mission. The present paper presents some results of preliminary trajectory studies which are useful in the design of nominal trajectories for the circumlunar mission. Part of the paper describes the results of a parametric study defining characteristics of trajectories which circumnavigate the Moon and return to the atmosphere of the Earth with re-entry conditions suitable for manned missions. Injection and midcourse guidance studies include an error analysis and calculations of the effects of guidance corrections at various points throughout two nominal trajectories. Finally, some considerations are given of the effect of the return point at the surface of the earth on the design of circumlunar trajectories.

INTRODUCTION

The design of a nominal trajectory for a manned circumlunar mission must take into consideration not only the various geometrical relations between the vehicle and the Earth- Moon system, but also the characteristics of the particular boost system and re-entry vehicle which will be used. The trajectory design must be consistent with the constraints imposed by such considerations as the launch site location, the desired point of return at the surface of the Earth, and the particular mission objectives. With the considerable number of variables involved, it is apparent that presentation of a few specific trajectories will not provide much information on the general characteristics of such trajectories.

The present paper describes some of the general characteristics of the circumlunar trajectories, as obtained from a parametric study, injection and midcourse guidance studies, and some additional considerations.

PARAMETRIC STUDY OF CIRCUMLUNAR TRAJECTORIES

The parametric study described in this section defines some of the characteristics of nominal trajectories which circumnavigate the Moon and return to the Earth with re-entry conditions suitable for manned missions. Most of the results given here are for two dimensional trajectories in the Earth-Moon plane, obtained by means of numerical integration of the equations of motion of the three-body problem of celestial mechanics. This simplification for a parametric study* allows presentation of the primary features of the trajectories without introduction of a large number of additional variables.

It is recognized at the outset, from preliminary studies, that there are two possible classes of return to Earth from a circumlunar mission; direct re-entry, in which the vehicle enters the atmosphere in the direction corresponding to the rotation of the Earth, and retrograde re-entry, in which the vehicle enters opposite to the rotation of the Earth. A typical trajectory in each class is shown, in the inertial coordinates, in Figure 1. Both of these trajectories pass within about 3,000 miles of the center of the Moon, or 2,000 miles from the surface, at the point of closest approach. It is noted that, although the retrograde re-entry trajectories have a higher injection energy, and thus shorter fight time to the vicinity of the Moon, its total time of flight for the mission is longer than that for the direct re-entry trajectories.

Circumlunar trajectories can be characterised by the energy of the trajectories, the minimum distance of the approach to the Moon, and the minimum distance of approach to the Earth on return to Earth. Relations between these parameters for some typical trajectories in the Earth-Moon plane are shown in Figure 2. The ordinate and abscissa are the minimum approach distance to the centers of the Moon (R_2 and Earth (R_1), respectively,

The double values of the abscissa correspond to the two classes of return of Earth. The energy of the trajectories is defined by the ratio of the injection velocity to the local parabolic velocity, V_1/V_p (V_p = 35,384 ft/sec at 300 miles altitude).

*Injection conditions used in the three-body calculations for the parametric study were: injection angle of 20°, injection altitude of 300 statute miles, variable injection velocity and lead angle, ψ (i.e., angle between the radius vector to the injection point and the radius vector to the Moon at time of injection). The Earth-Moon distance was taken as the mean distance, 238,857 statute miles.

Trajectories with perigee distance less than about 4,000 miles will return to the Earth and re-enter the atmosphere, but only those trajectories with perigee distance of approximately 4,000 miles will have re-entry conditions suitable for maned lunar missions. As the energy of the trajectory increases, the spacecraft must pass closer to the Moon to obtain acceptable direct re-entry trajectories.

The situation is somewhat different for retrograde re-entry trajectories in that a "tangency" condition occurs. This tangency condition is of interest in that, for this condition, the return perigee distance is relatively insensitive to changes in the distance of approach to the Moon. It is clear that by adjusting the energy of the trajectory, the tangency condition can be situated at values of perigee distance corresponding to a desired re-entry corridor, thereby decreasing the sensitivity of perigee distance to Moon approach distance. There is no such tangency condition for the direct re-entry trajectories.

Acceptable circumlunar trajectories occur in a narrow range of energy, or injection velocity. The difference in injection velocity between the lowest and highest energy trajectories shown in Figure 2 is only 0.0028 V_p, or about 100 fps.

Figure 3 shows some design parameters for circumlunar trajectories. The ordinate is R_2, the minimum approach distance from the center of the Moon, and the abscissa is the lead angle ψ, the angle between the radius vector to the injection point and the radius vector to the Moon at time of injection. Combination of parameters between the two boundaries results in trajectories which return to the Earth, but only those combinations near the boundaries give acceptable re-entry corridors. Note that for the retrograde re-entry class, the sensitivity of perigee distance to changes in ψ can be decreased by proper choice of the energy and the Moon approach distance.

Figure 4 shows the position with respect to the Moon of the point of closest approach. (A paper by Lieske*, containing some work similar to this has recently come to the attention of the authors). Again, positions and energies between the boundaries define trajectories which will return to the Earth, but only those near the boundaries give acceptable re-entry corridors. Note that the direct re-entry boundary lies nearly along the Earth-Moon line, so that trajectories in this class, with perigee distance ~4,000 miles, are expected to be nearly symmetrical on the outbound and return legs.

Figure 5 is a summary plot relating to the total flight time of the trajectory, the injection velocity ratio, and the minimum approach distance to the center of the Moon, for trajectories which return to the Earth with perigee distance of 4,000 miles. For corresponding values of Moon approach distance, the retrograde re-entry trajectories have longer total flight times and require higher injection velocity ratios than do the direct re- entry trajectories. From the figure, the minimum flight time for a circumlunar mission is about 5 ¾ days, for a trajectory which skims the surface of the Moon. The time of flight for trajectories which pass at about 3,000 miles from the surface of the Moon increases to 7.2 to 8.8 days, for direct and retrograde re-entry, respectively.

---

* "Circumlunar Trajectory Studies," by H. A. Lieske, Rand Report F-1441, June 1958

GUIDANCE CONSIDERATIONS

As part of the parametric study, some trajectory calculations have been made to give an indication of guidance accuracies required for the mission, and the effects of midcourse corrections at various points throughout the trajectory.

The results of a first-order error analysis giving the change in return perigee distance as a function of errors in the nominal injection conditions are shown in Figure 6. The top part of the figure gives the partial derivative of perigee distance with respect to injection angle γ, in miles per degree, and the bottom part the partial of perigee distance with respect to injection velocity in miles per foot per second, both as a function of Moon approach distance for the nominal trajectory. The retrograde re-entry curves have derivatives with signs opposite to those on the scales, hence the negative notation, but are plotted this way for easy of comparison.

The plots show that the sensitivity of the perigee distance to these injection errors is a strong function of how closely the trajectory approaches the Moon, as expected.

The interesting feature of the plots is that the sensitivity curves for the retrograde re-entry trajectories go through zero for particular nominal trajectories, while such conditions do not occur for the direct re-entry trajectories. (The point for which $\left(\frac{\partial R_1}{\partial \gamma}\right)_{V_1}$ goes to zero is the tangency point as discussed in Figure 2.)

As indicated on the figure, the derivatives with respect to velocity and injection angle do not both go through zero at the same values of moon approach distance, so these two zero points correspond to two different nominal trajectories. If it is considered that the injection velocity can be controlled better than the injection angle, then the nominal trajectory for which $\left(\frac{\partial R_1}{\partial \gamma}\right)_{V_1}$ is zero might be used and vice versa. It is noted that the sensitivities of the retrograde trajectories to errors in the initial conditions are smaller than those of the direct trajectories within most of the R_2 range shown in the figure.

For an indication of midcourse guidance requirements, the effects of midcourse velocity corrections were determined at several points along each of the two nominal trajectories. The nominal trajectories chosen were a direct and a retrograde re-entry trajectory, each of which had a minimum approach distances to the center of the Moon of about 3,000 miles, such that for the retrograde re-entry trajectory the derivative $\left(\frac{\partial R_1}{\partial \gamma}\right)_{V_1}$ (as defined in Figure 6) was zero. These are the two trajectories which are shown in Figure 1.

It is of some interest to note the injection accuracy requirements for these two nominal trajectories for accomplishing the circumlunar mission and returning to the Earth within a 40-miles corridor without midcourse corrections. The allowable errors in injection velocity and injection angle are:

For direct re-entry:
$\Delta V_i = 0.017$ ft/sec
$\Delta \lambda = 0.0016$ deg

For retrograde re-entry:
$\Delta V_i = 0.017$ ft/sec
$\Delta \lambda = 0.0274$ deg

The amount of midcourse correction required will, of course, depend on the accuracy of injection into the lunar trajectory, and will be a function of the characteristics of the particular launch vehicle. From the above result, it can be expected that the correction required in return perigee distance will be less for the retrograde re-entry trajectory.

Some results of the midcourse velocity correction calculations are given in the table shown in Figure 7. For velocity corrections applied at positions along the trajectory defined by the time of flight from the injection point and corresponding distance from the Earth R_g, the table shows

the magnitude of the changes in return perigee distance and total time of flight, for velocity corrections applied in the particular direction for maximum change in each of these quantities. The angle θ is the angle between the direction of midcourse velocity corrections for which perigee distance and time of flight changes are maximums. The last two columns give the magnitudes of the changes in perigee distance and in time of flight when the velocity correction is applied in such a direction that the other parameter is unchanged.

On the outbound leg of the trajectory, the values of $\left(\frac{\partial R_1}{\partial \gamma}\right)_{Max}$ for the direct re-entry indicate large changes in perigee distance for a given magnitude of applied velocity. But because of the small values of θ, it is not possible to utilize this high sensitivity without also introducing large changes in time of flight. Also, because of the small values of θ, an error in the direction of application of the velocity correction could introduce a fairly large error in either perigee distance or time, which must be accounted for by a later correction.

For the retrograde trajectory, on the other hand, the table indicates that the time of flight sensitivity is high, and that any early corrections to be made in the time of flight must be applied quite accurately. This difficulty could be alleviated by making time of flight corrections later on in the trajectory where the sensitivity is decreased.

The sensitivities of the midcourse correction derivatives on the return leg of the circumlunar flight are practically identical for the direct and retrograde re-entry trajectories, indicating no particular advantage of one class with respect to the other in the region.

As a general conclusion of this discussion, it can be said that the retrograde re-entry trajectory appears to have some overall advantages with respect to the direct re-entry trajectory from guidance and control considerations. It is not at all certain, however, that such advantages outweigh the inherent disadvantages of the retrograde re-entry trajectories, such as increased re-entry velocity and consequent increase in heating protection required, somewhat larger total time of flight, and, possibly, more difficult tracking considerations during the terminal phase of the trajectory.

CONSIDERATIONS OF RETURN POINT OF DESIGN OF CIRCUMLUNAR TRAJECTORIES

Specifications of a desired latitude and longitude of the touchdown point on return from a circumlunar mission will exert a considerable influence on the design of the trajectory. In this section, consideration is mainly directed to obtaining a desired latitude, and to show how this will affect the preliminary trajectory design. Obtaining the desired longitude will be primarily a matter of proper choice of total flight time and accurate control of the flight time by corrections applied directly to the trajectory. Both latitude and longitude are, of course, affected by the vehicle maneuverability after entering the Earth's atmosphere, and particularly by the range from the re-entry point to the touchdown point.

From consideration of typical trajectory characteristics, it is possible to determine approximate trajectory requirements for obtaining a desired latitude of the touchdown point on the Earth's surface.

Consider the schematic drawing in Figure 8. For a return point in the northern hemisphere, the trajectory is designed so that the closest approach to the Moon occurs at, or close to, maximum negative declination. The Earth-Moon line at this time of closest approach is shown on the top of the figure. To first order, this line can be considered to be the line of nodes of the trajectory plane and the Earth-Moon plane. A projection of this line through the Earth to the northern hemisphere intersects the Earth's surface at a point denoted by L. This is shown also in the lower part of the figure.

By specifying conditions on the position of the trajectory relative to the Moon as the vehicle approaches the Moon, the inclination of the return trajectory plane to the Earth-Moon plane, ρ, can be controlled. Controlling the inclination of the return trajectory plane to the Earth-Moon plane gives a means for obtaining a desired latitude of the touchdown point.

We now consider the location of the conic perigee of the return ellipse and the atmospheric re-entry point relative to the line of nodes of the trajectory and Earth-Moon planes. From consideration of the return phase of typical direct re-entry circumlunar trajectories we determine that the angular arc between the conic perigee and the node line is of the order of 7½°. This can be illustrated schematically by drawing an arc with this angular radius centred at the node point in the surface of the Earth. The conic perigee can thus be located at positions along this arc as determined by the value of the trajectory plane inclination, ρ, as shown on the sketch in Figure 8.

In order to locate the re-entry point, we note that for a vehicle with some lift capability (for instance, with L/D of the order of ½), the $10G_{MAX}$ corridor at 400,000-foot altitude is defined by re-entry angles in the range of about -8° to -5°. For purposes of illustration, we take an average value of $\gamma_{Re} = -6.5°$. For this energy trajectory, the true anomaly is approximately twice the re-entry angle. Therefore, the angle between the re- entry point and the conic perigee is about $2\gamma_{Re}$ or -13°.

Combining the angular values obtained for the location of the re-entry point relative to the conic, and the location of the conic perigee relative to the node, we obtain a value for the location of the re-entry point relative to the node of the order of 20½°. The location of the re-entry point can then be illustrated schematically by drawing an arc with radius of 20½° and centered at the node point. By reference to the schematic drawing in Figure 8, it can be seen that the latitude of the re-entry point can be less than, equal to, or greater than the declination of the Moon at closest approach, depending on the angle ρ, the inclination of the trajectory plane to the Earth-Moon plane.

To locate the latitude of the touchdown point, consideration must be given to the range of the vehicle from the re-entry point to the touchdown point. It is first noted that for the typical example of 20½° arc between the re-entry point and the node, if the range from the re-entry point to the touchdown point also happens to be 20½° (≈1230 nmi), then the latitude of the touchdown point will be the latitude of the node line, irrespective of the inclination angle ρ. This range of 20½° thus represents a point of division, such that ranges greater or less than this value will result in latitudes of the touchdown point greater or less than the latitude of the node, depending on the value of ρ.

From consideration of the trajectories of typical re-entry vehicles*, it is noted that a typical value of minimum range from the re-entry point to the touchdown point is of the order of 20 to 25° (1,200 to 1,500 nmi). These values are obtained for the steepest re-entry angle, and considerably greater ranges are required if the re-entry is made at the shallower angles. Allowance must be made, of course, for accommodation of all re-entry angles defining the acceptable corridor width. It is thus clear that the design range from the re- entry point to the touchdown point must be considerably greater that the 20½° value discussed above.

In view of the above discussion, some conclusions can be drawn. If the desired latitude of the touchdown point is in midcontinental United States (≈ 40 N), then this latitude is greater than the maximum declination of the Moon for any year (maximum declination ≈ ±28.5°).

The designed range from the re-entry point to touchdown will be greater than 20.5°, so that to achieve the desired latitudes, ρ must be positive as defined in Figure 8. The trajectory thus passes from southwest to northeast with respect to the surface of the Earth, with an azimuth heading north of east at the touchdown point.

Some calculations have been made to indicate the relations between the latitude and azimuth heading of the vehicle at the touchdown point, the inclination of the trajectory plane to the Earth-Moon plane (ρ), the inclination of the trajectory plane to the equatorial plane (i), and the range from the re-entry point to the touchdown point. Some typical results are shown in Figure 9. These results are for approaching the Moon when the Moonis at a maximum declination of -28°, and for direct return trajectories with a re-entry angle of 6.5°.

---

*Parameter Sensitivity and Guidance Viewpoints For Circumlunar Flight and Return, R.F. Hoelker, N.J. Braud, O.C. Jean, and A.J. Schwaniger, Marshall S.F.C., Rept. MNN-M-Aero-3-60, Aug 1960

The calculation shows that the latitudes of the order of 40°N can be obtained with ranges of about 40° or greater, and with inclinations of the trajectory plane to the equatorial plane of about 40° to 60°. The corresponding azimuth headings at touchdown are indicated on the figures. It is found that considerations as discussed above, and similar calculations for retrograde re-entry trajectories, are of help in preliminary design of circumlunar and lunar return trajectories for which touchdown conditions on the surface of the Earth are specified.

SYMBOLS

R_1	conic perigee distance, measured from center of Earth, miles
R_2	distance of closest approach to center of Moon, miles
$\frac{R_g}{D}$	vehicle distance from center of Earth as ratio of distance from Earth to Moon
T	total flight time from injection to conic perigee, hr
$\frac{V_i}{V_p}$	ratio of injection velocity to local parabolic velocity (Vp=35,384 fps at altitude of 30 statute miles.
V	midcourse velocity corrections, fps
γ	injection angle, angle between local horizontal and velocity vector, deg
γ_{Re}	re-entry angle, deg
θ	angle between directions of midcourse corrections for which changes in perigee distance and time of flight are maximum, deg
θ_i	inclination of return trajectory plane to equatorial plane, deg
i	inclination of return trajectory plane to Earth-Moon plane, deg
ρ	inclination of return trajectory plane to Earth-Moon plane, deg
ψ	lead angle, angle between the radius vector to the injection point andthe radius vector to the position of the Moon at time of injection, deg

FIGURES

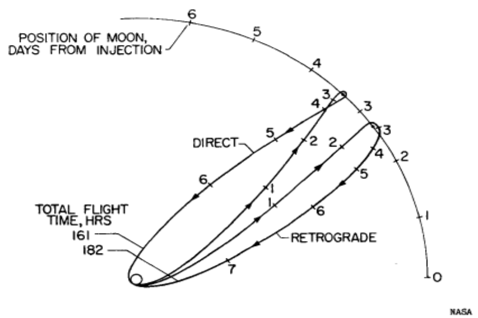


Figure 1.- Typical circumlunar trajectories.

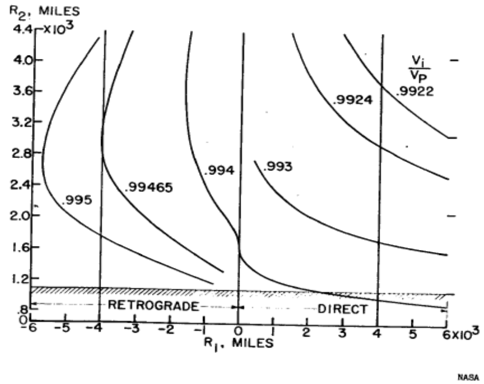


Figure 2.- Parametric study results.

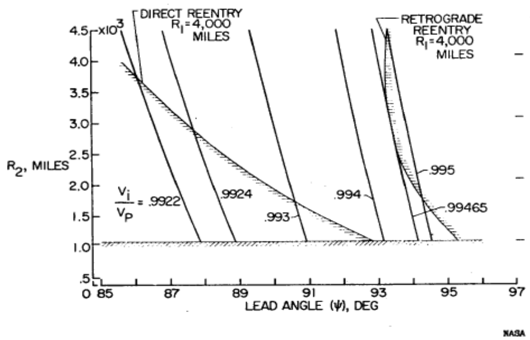


Figure 3.- Design parameters.

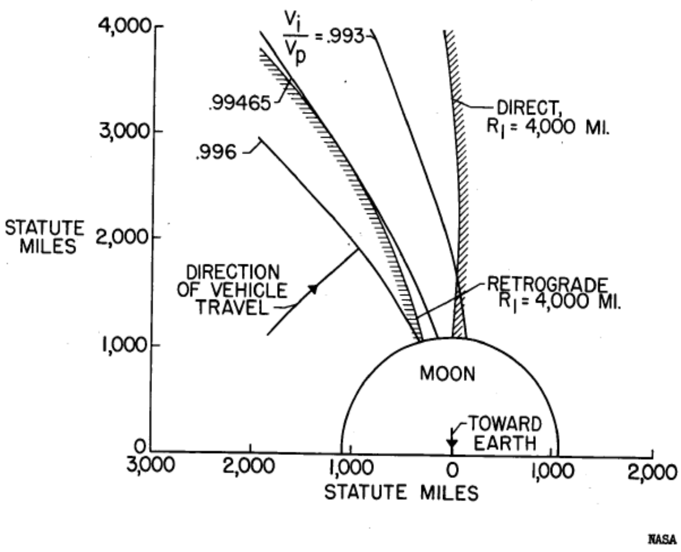


Figure 4.- Position of vehicle at closest approach.

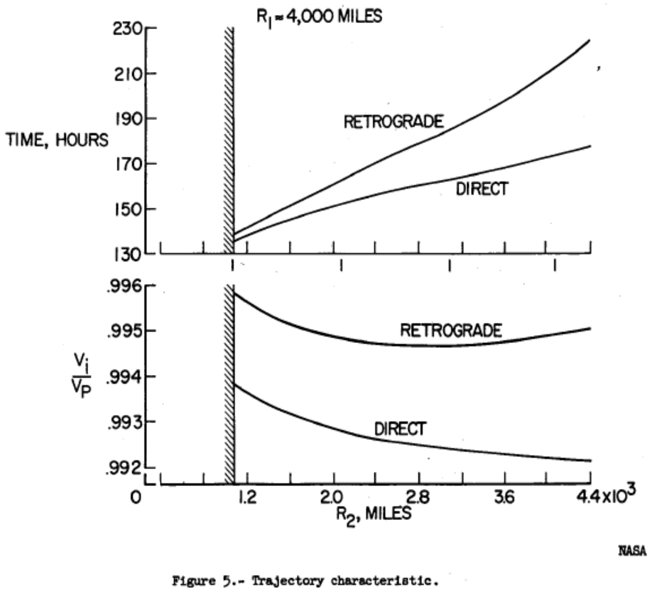
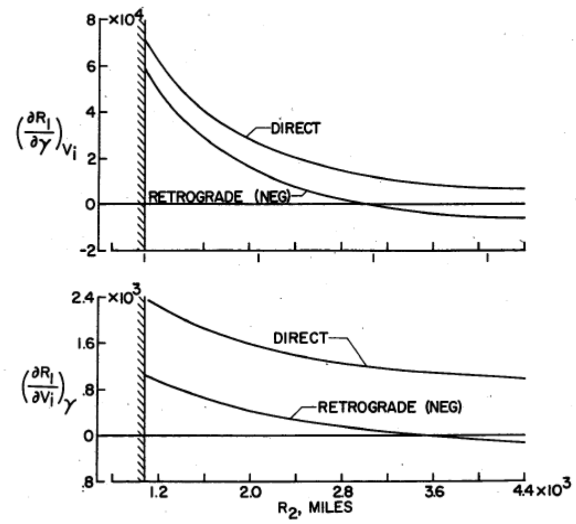


Figure 5.- Trajectory characteristic.



NASA

Figure 6.- Results of error analysis.

MIDCOURSE GUIDANCE RESULTS

TIME FROM INJECTION, HRS.	$\frac{R_E}{D}$	$\left(\frac{\partial R_I}{\partial V}\right)_{MAX.}$ MILES/FPS	$\left(\frac{\partial T}{\partial V}\right)_{MAX.}$ HRS/FPS	θ, DEG	$\left(\frac{\partial R_I}{\partial V}\right)_{T = CONST.}$ MILES/FPS	$\left(\frac{\partial T}{\partial V}\right)_{R_I = CONST.}$ HRS/FPS
			DIRECT REENTRY			
9.8	.307	282	.503	3.9	19.2	.034
26.5	.568	174	.322	7.8	23.6	.044
81.2	1.012	25	.014	55.9	21.8	.012
96.6	.920	12.4	.012	66.1	11.3	.010
153.2	.278	3.7	.001	64.8	3.3	.0009
			RETROGRADE REENTRY			
10.2	.318	25.5	.991	35.4	14.8	.574
23.4	.548	18.7	.696	52.3	14.8	.550
64.9	1.012	20.7	.047	37.2	12.5	.029
106.1	.927	12.6	.015	73.8	12.1	.014
173.0	.302	4.0	.0008	62.2	3.6	.0007

NASA

Figure 7.- Midcourse guidance results.

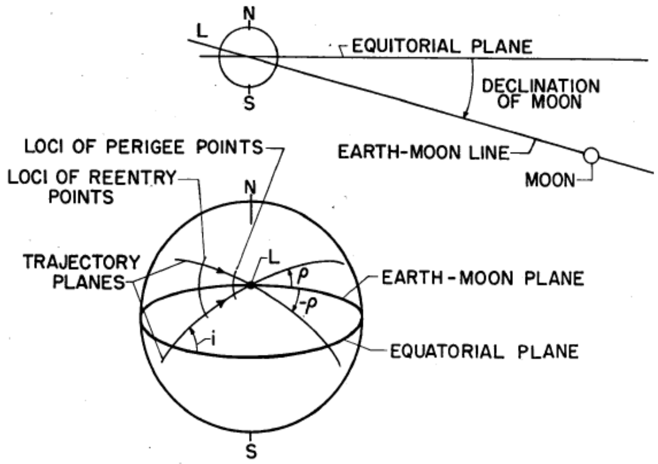


Figure 8.- Trajectory design considerations.

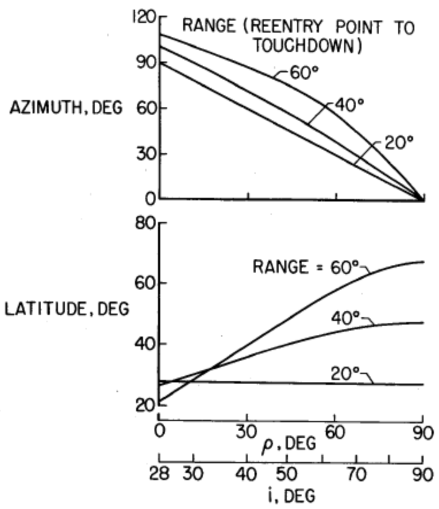


Figure 9.- Touchdown point characteristics.